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ProtaStructure Design Guide

Column Design Verification by (N-M) Interaction Diagram

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Column Interaction Diagram

Objective

This document aims to outline the steps to verify reinforced concrete column design using the capacity curve, commonly referred to as N-M (Axial-Moment) Interactive Diagram.

The N-M Interaction Diagram will be derived using empirical formulas of concrete stress block. This is then compared to the same capacity curve generated by the ProtaStructure column design to verify the validity and adequacy of its column design.

This method can be used to check and verify the adequacy of any column design program. The capacity curve generated by any column design application should tally with the empirical curve derived from the first principles.

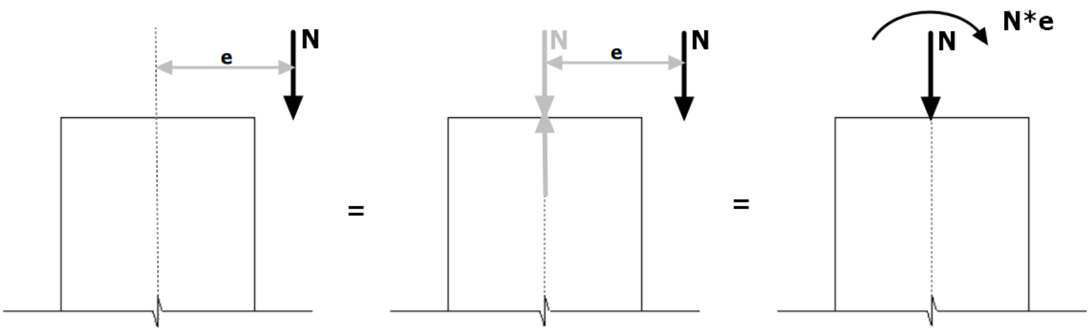
Introduction

Reinforced columns and walls are subjected to combined axial loads and bending moments as part of a 3D building frame behaviour. Due to monolithic construction, the column and beam joints should be assumed to be fixed for reinforced concrete buildings. Hence, all columns and walls will be subjected to axial and moment forces because of gravity and lateral external loads.

A reinforced concrete column's capacity depends on the coincident axial load and the moment, i.e., they are not mutually exclusive. An interactive diagram is a capacity curve that can be plotted to completely describe the strength of the reinforced column in terms of its axial and moment capacity. Hence, using the term capacity curve & N-M interaction diagram interchangeably is common.

Therefore, an interaction diagram can be used to check against the design load combinations and determine a reinforced structural member's acceptable moment and axial capacities.

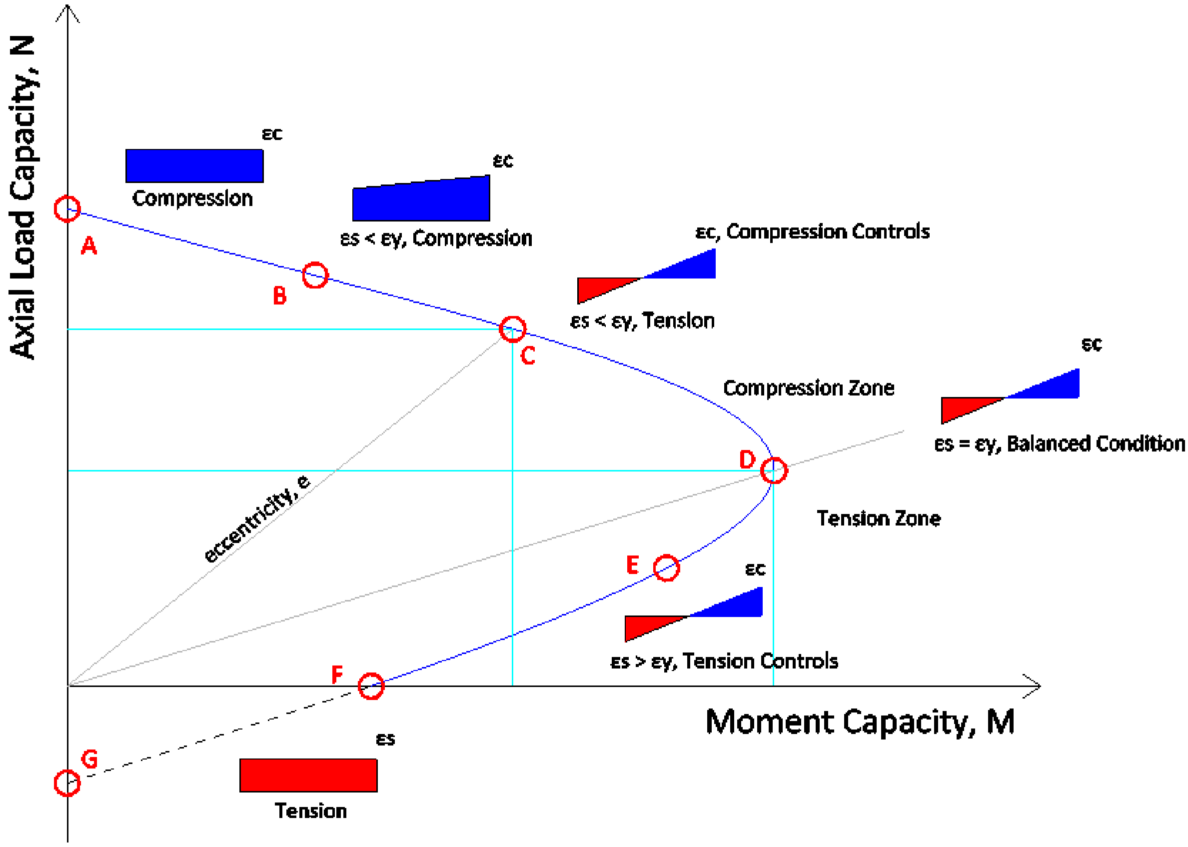
If axial load N is applied with eccentricity, e (distance away from centroid), it creates a pure moment, N*e, and to be combined with the initial axial load, N.



As the eccentricity "e" changes from the section centroid, behaviours of a column/wall and stress distribution in the section will be generated differently. These are used to plot the Axial Load to Moment (N–M) Interaction curve.

In reinforcement concrete buildings, moments are naturally generated due to the fixity of the joints even under gravity load, not necessarily due to eccentric eccentricity alone.

Axial Load to Moment (N-M) Diagram Explained:



Case A: Pure axial compression

- The column can only achieve this maximum compression capacity if there are zero coincident moments. This capacity can never be achieved in reality, as the small moment will always exist.

Case B: Compression with minor bending due to small eccentricity.

- The axial capacity starts to decrease with an increase in the bending moment.
- Ground-level columns are likely to be at this part of the curve as it is subject to high axial load.

Case C: Compression controls

- The axial capacity decreases rapidly with the increasing moment.
- Column subjected to compression and tension whereas the stress in steel at tension side, $f_{st,}$ is less than the yield stress, f_{yk}.

Case D: Balanced Condition

- This is the maximum moment capacity, provided the coincident axial value is exactly value D.
- Compression concrete strain, ε_c reaches the maximum strain limit, and steel strain, ε_s is equivalent to the steel strain limit, ε_y.

Case E: Tension controls.

- Both the axial & moment capacity decreases after the balanced point.
- For example, a roof column will likely be at this part of the curve due to low axial loads. Hence, roof columns have a lower moment capacity than ground-level columns.
- Column subjected to compression and tension whereas the stress in steel at tension side, fst is greater than the yield stress, fyk.

Case F: Pure Flexure

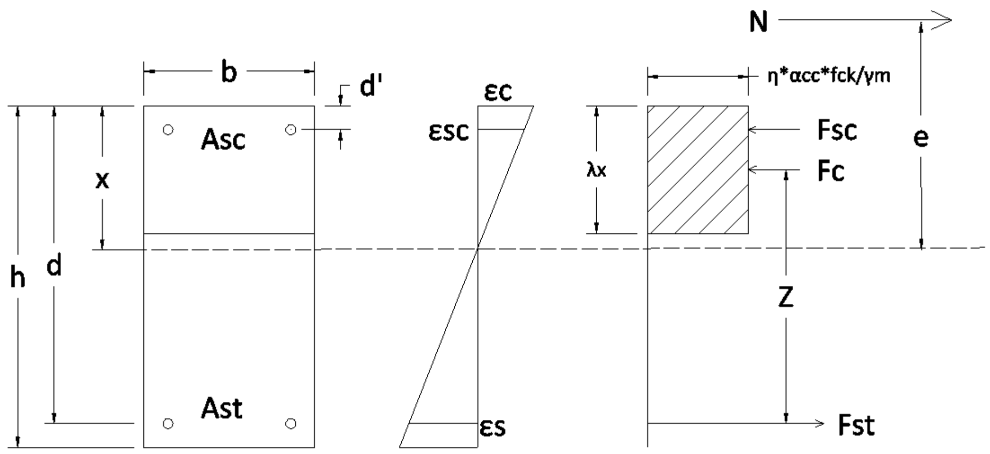
- The column is subjected to pure moment only & zero axial force.

Case G: Pure axial tension

- Column subjected tension force only.
- This case happens to buildings subjected to rotational force where some columns are in compression, and some are in tension.
- This maximum tension capacity can never be achieved as a small bending moment will always exist.

Stress Block

All formulas will be extracted and derived from this stress block.



Notation Based on the Stress Block

Symbols	Definition
A_{sc}	Reinforcement area at extreme compression side
A_{st}	Reinforcement area at the extreme tension side
b	Member width
d	Member depth
d'	Distance from the compression face to the center of the first layer rebars
e	Eccentricity
F_c	Force at compressive side by concrete
F_{sc}	Force at compressive side by steel
F_{st}	Force at tension side by steel
f_{yk}	Rebar steel grade
h	Member height
N	Axial force
x	Distance between compression face to the neutral axis
Z	The lever arm of Internal Forces, distance from F_c to F_{st}
α_{cc}	Coefficient - eqn. 3.15
ε_c	Strain in concrete at the compressive side
ε_s	Strain in steel at the tension side
ε_{sc}	Strain in steel at compressive side
λ	Reduction Factor
η	Effective Strength
Υ_m	Concrete Modification Factor

Formula:

Reinforcement area at extreme tension side:

$$Ast = no.bar\,.\frac{\pi.\varphi^2}{4}$$

Reinforcement area at extreme compression side:

$$Asc = no.bar.\frac{\pi.\varphi^2}{4}$$

Strain in steel at Yield Strength:

$$\varepsilon y = \frac{fyk}{Es} = 0$$

Strain in steel at tension side:

$$\varepsilon_s = \varepsilon_c.\left(\frac{d-x}{x}\right)$$

Strain in steel at compression side:

$$\varepsilon_{sc} = \varepsilon_c.(\frac{x-d'}{x})$$

Tensile stress at tension side:

$$fs = \varepsilon_s.Es$$

Tensile stress at compression side:

$$fsc = \varepsilon_{sc}.Es$$

Compressive stress block depth:

$$Eurocode\ 2\ cl.2\ 3.1.7(3):\quad RF.\lambda x$$

Forces at compression side:

By concrete: $Fcc = fcd.(RF.\lambda x).b.\eta$

By steel: $Fsc = Asc.[\max((min(fy, \varepsilon sc.E), -fyd) - (fcd)]$

Force at tension side:

By steel: $Fst = Ast.min(fy, \varepsilon st.E)$

Summation of forces:

$$\sum F = 0;$$
$$Fst + N = Fcc + Fsc$$
$$N = Fcc + Fsc - Fst$$

Summation of Moment about F_{st}:

$$N.e.\left(\frac{h}{2} - d'\right) - Fcs.(d - d') - Fcc.(d - \frac{x}{2}) = 0$$

Eccentricity about F_{st}:

$$E = \frac{Fcs.(d - d') + Fcc.\left(d - \frac{\lambda x}{2}\right)}{N} - (h/2 - d')$$

Ultimate Moment Capacity:

$$M = N.e$$

Ultimate Moment Capacity Limit (**British Standard Only – BS8110-1, Eqn. 38**):

$$N = 0.4.f_{cu}.A_c + 0.75.A_s.f_{yd}$$

Example (Eurocode):

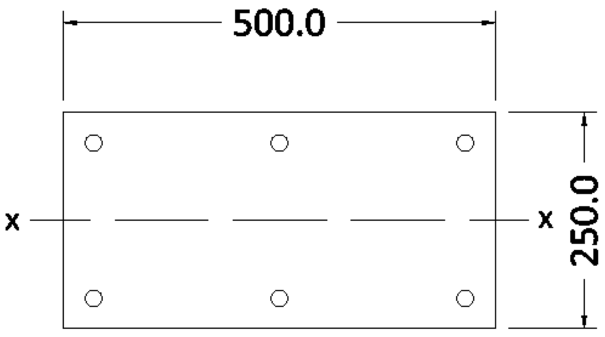
Take column 1C5 as an example from **"Quick Start Guide Concrete Complete"**.

Column 400mm x 200mm, concrete grade C30/37, steel grade 500 (Type 2),

Main Bar diameter= **H13**, link bar diameter= **H10**, cover = **35mm**.

Take the first three design loads from Design Report in Interactive Design and design based on N and M22.

Case	Axial Load	Without imperfection and second-order effect	With imperfection and second-order effect
	N (kN)	M22 (kNm)	**M22 (kNm)**
1	**999.47**	23.7	**28.04**
2	**960.04**	24.6	**28.78**
3	**848.89**	15.1	**18.83**



Factor:	Symbol:	Value:
Coefficient - eqn. 3.15	αcc	0.85
Effective Strength	η:	1.0
Reduction Factor	λ:	0.8
Reduction Factor - cl.3.1.7(3)	RF:	0.9
Max. Strain of Steel	εc:	0.0035
Steel Modulus	Es:	200,000 N/mm^2
Concrete Modification Factor	γm:	1.5
Rebar Modification Factor	γms:	1.15

Plastic Centroid, yp = 200mm

Calculation:

Case G: Pure Tension from Steel

$$Total\ Axial\ Load, N = -Ast.fyd$$
$$N = -346.25kN$$

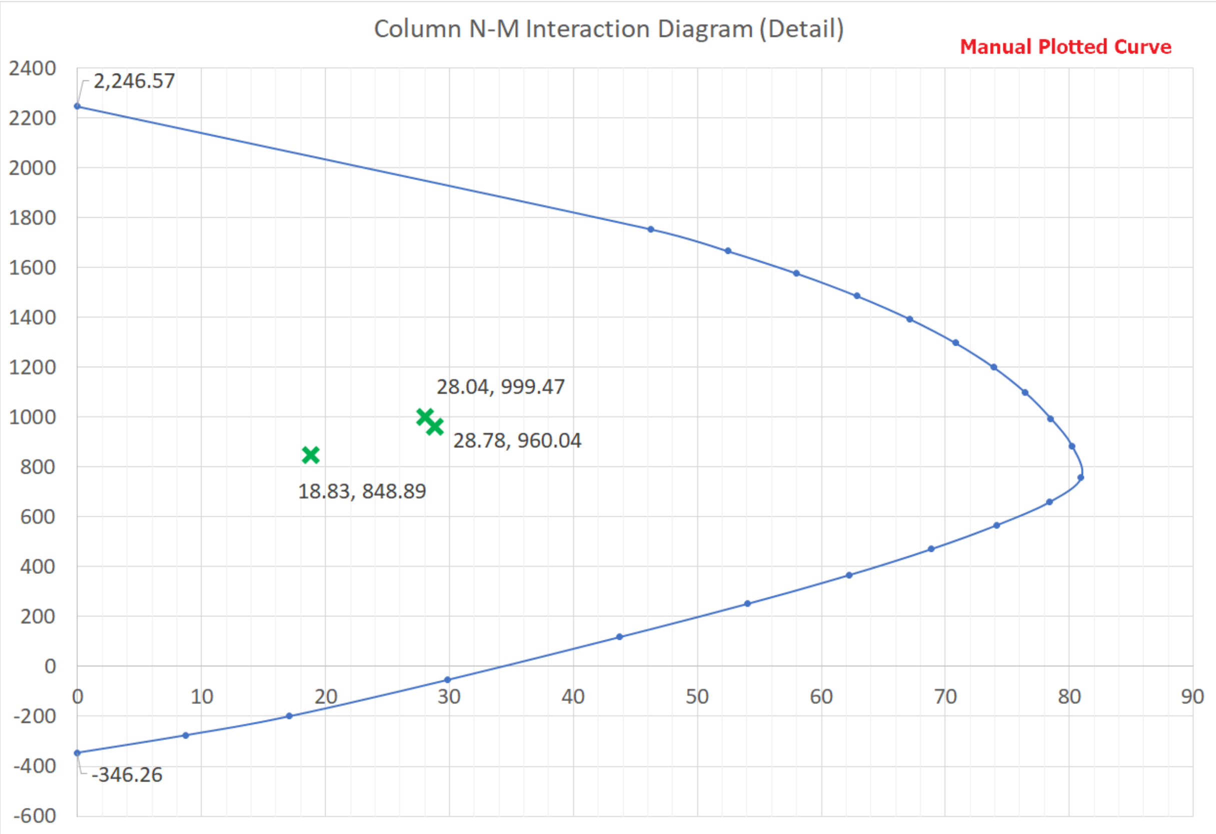
Case A: Pure Compression:

$$Total\ Axial\ Load, N = Fsc + Fcc$$
$$N = (346.25 + 1{,}900.32)kN$$
$$N = 2{,}246.57\ kN$$

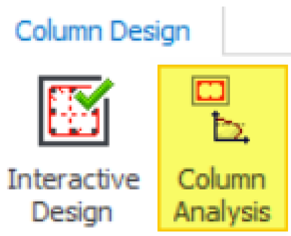
The sample size for x is 20 in this example. Increase the size to have higher precision of the result.

$$x\ (mm) = \begin{pmatrix} 20 \\ 40 \\ 60 \\ 80 \\ 100 \\ 120 \\ 140 \\ 160 \\ 180 \\ 200 \\ 220 \\ 240 \\ 260 \\ 280 \\ 300 \\ 320 \\ 340 \\ 360 \\ 380 \\ 400 \end{pmatrix} N, ult\ (kN) = \begin{pmatrix} -276.53 \\ -200.03 \\ -54.46 \\ 117.74 \\ 251.66 \\ 366.44 \\ 470.28 \\ 567.29 \\ 659.74 \\ 758.23 \\ 884.20 \\ 994.24 \\ 1099.11 \\ 1199.93 \\ 1297.51 \\ 1392.45 \\ 1485.22 \\ 1576.19 \\ 1665.63 \\ 1753.78 \end{pmatrix} M, ult\ (kNm) = \begin{pmatrix} 8.68 \\ 17.10 \\ 29.82 \\ 43.74 \\ 54.08 \\ 62.25 \\ 68.85 \\ 74.18 \\ 78.42 \\ 80.97 \\ 80.22 \\ 78.52 \\ 76.44 \\ 73.88 \\ 70.80 \\ 67.15 \\ 62.90 \\ 58.01 \\ 52.47 \\ 46.26 \end{pmatrix}$$

Based on the output from manually potted curve, say using Excel program, as shown below.



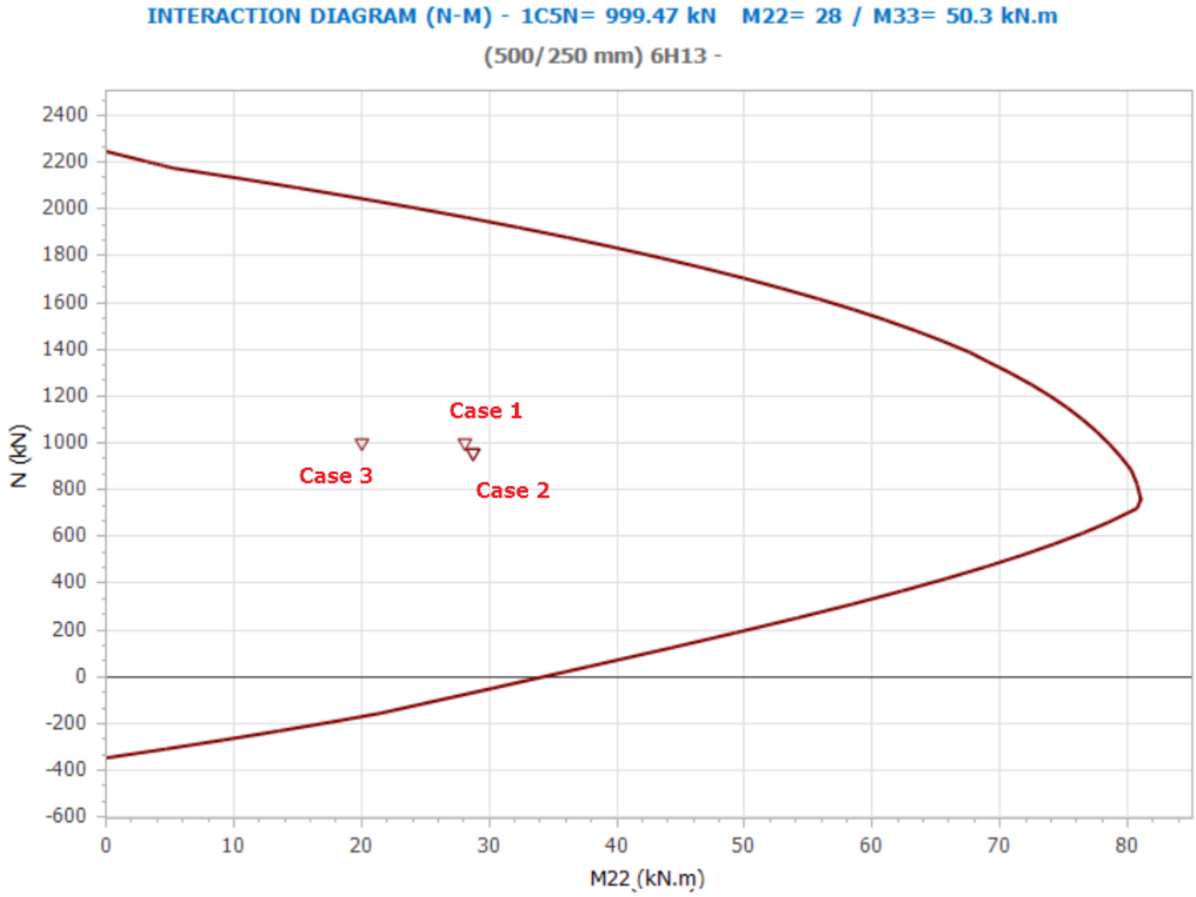
In ProtaStructure, the interaction diagram can be displayed in "Column Analysis" in the Interactive Column design after a successful design (as shown below).



Design Loads: **Design Report from Interactive Design**

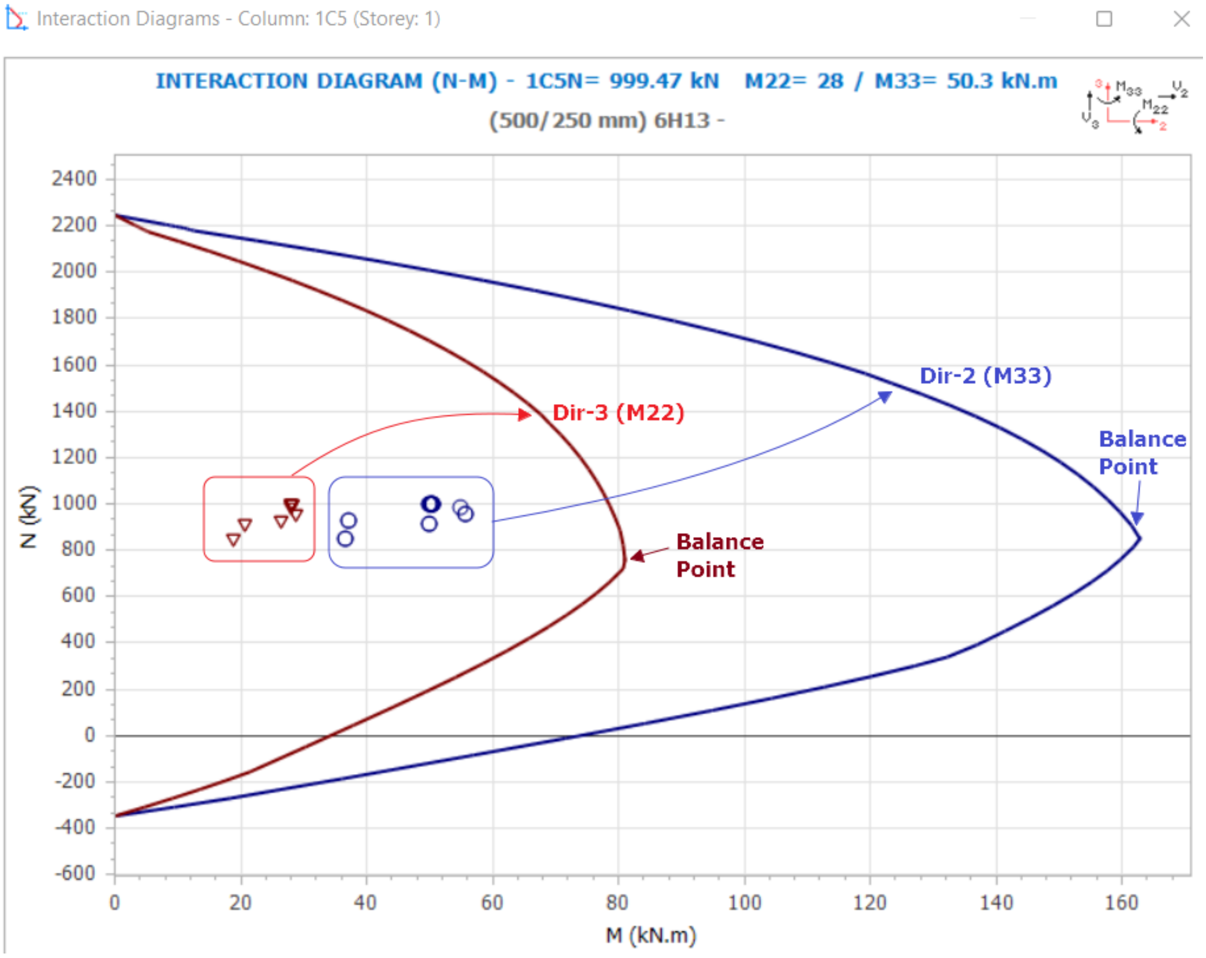
No	N (kN)	M22 (kN.m)
1	999.47	28.04
2	960.04	28.78
3	848.89	18.83

The curve for direction 3 (M22) about "plane X" from ProtaStructure is shown below:



The interaction diagram generated is identical to the manually plotted diagram derived using empirical formulae. Hence, we can conclude that the N-M interaction Diagram result in ProtaStructure is correct and adequate.

Curve for both direction 2 (M33) & 3 (M22) is shown below:



Both directions (2 & 3) can be verified using the same formulae. When the design loads fall within the curve, the column section size and applied steel rebars are adequate. The governing design will be the direction with load combinations closer to the curve. For this case, the governing curve is the dir-3 (M22).

If any design loads from either direction surpass the curve, the column design fails.

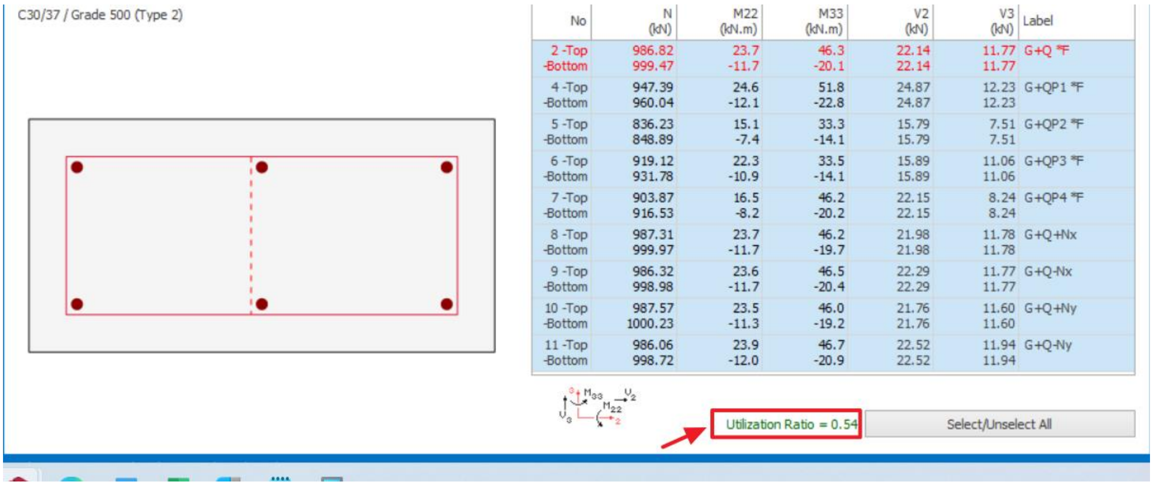
If the rebar sizes, rebar numbers or the size of the column increases, the boundary of the curve will increase, i.e. both the axial and the moment capacity increase; vice-versa.

ProtaStructure auto-design will attempt to increase the number and sizes of rebars to the maximum allowable as defined in the column design settings while keeping the column size constant. If this fails, the column will be marked as failed, with the message “Section Insufficient”.

Alternatively, the user may refer to the curve and manually reduce/increase the rebar or column section size for each direction. Then recheck the capacity curve to ensure all the design points are within the capacity curve in each direction respectively.

Column Utilisation Ratio

After “Interactive Design” has been performed, the Utilisation Ratio will be shown at the bottom of the dialogue. This utilization ratio shows the design efficiency of the column.



No	N (kN)	M22 (kN.m)	M33 (kN.m)	V2 (kN)	V3 (kN)	Label
2 -Top	986.82	23.7	46.3	22.14	11.77	G+Q *F
-Bottom	999.47	-11.7	-20.1	22.14	11.77	
4 -Top	947.39	24.6	51.8	24.87	12.23	G+QP1 *F
-Bottom	960.04	-12.1	-22.8	24.87	12.23	
5 -Top	836.23	15.1	33.3	15.79	7.51	G+QP2 *F
-Bottom	848.89	-7.4	-14.1	15.79	7.51	
6 -Top	919.12	22.3	33.5	15.89	11.06	G+QP3 *F
-Bottom	931.78	-10.9	-14.1	15.89	11.06	
7 -Top	903.87	16.5	46.2	22.15	8.24	G+QP4 *F
-Bottom	916.53	-8.2	-20.2	22.15	8.24	
8 -Top	987.31	23.7	46.2	21.98	11.78	G+Q+Nx
-Bottom	999.97	-11.7	-19.7	21.98	11.78	
9 -Top	986.32	23.6	46.5	22.29	11.77	G+Q-Nx
-Bottom	998.98	-11.7	-20.4	22.29	11.77	
10 -Top	987.57	23.5	46.0	21.76	11.60	G+Q+Ny
-Bottom	1000.23	-11.3	-19.2	21.76	11.60	
11 -Top	986.06	23.9	46.7	22.52	11.94	G+Q-Ny
-Bottom	998.72	-12.0	-20.9	22.52	11.94	

This utilization ratio will be based on three checks and shown only the governed check. The checks can be found in the “Design Report” in the “Interactive Design”. The three checks are:

1. Required steel area over provided steel area.
2. Design axial Load, Nd, over the maximum axial load capacity of the column. (This check only applies to EC8 and BS8110.)
3. Design moment, Md, over maximum moment capacity, Msup, of the column.
   a. M_d = Design moment
   b. M_{sup} = Maximum column moment capacity

   The formula to calculate Md and Msup is as below:

$$Md = \sqrt{(design\ M22)^2 + (design\ M33)^2}$$

$$Md = \sqrt{50.3^2 + 28^2} = \mathbf{57.57kNm}$$

$$Msup = \sqrt{(max\ M22)^2 + (max\ M33)^2}$$

$$Msup = \sqrt{93^2 + 52^2} = \mathbf{106.6kNm}$$

The moment capacity ratio in item (3) can be explained by referring to the Interaction Diagram (M22-M33), as shown in the figure below.

The above is the moment capacity curve expressed in moments in two directions, respectively (M22, M33) for a given axial load. This curve is plotted for the case N = 999.47 kN, obtained for critical load combination 2.

The shape and boundary of the curve will change with axial load in the same way that the N-M interaction diagram behaves. Similarly, the moment design point should be within the boundary of the curve to prove the column passes.

```
UTILIZATION RATIOS

    As (Required/Supplied)           = 125.00 / 796.39                  = 0.16
    N  (Nd)                          = 999.47
    M (Md/Msup)                      = 57.55 / 106.80                   = 0.54
```

Based on the three utilization ratios of column 1C5, M_d/M_{sup} ratio has the highest figure. Hence, we can conclude that the column is governed by the bending moment.

Example (British Standard):

Another quick and simple example of the BS8110 reinforced concrete column:

Column 400mm x 400mm, concrete grade 30, steel grade 460 (Type 2),

Main Bar diameter= T12, link bar diameter= T10, cover = 30 mm.

Apply dead load 180 kN vertically and 15 kN horizontally (dir-y) at the top of the column.

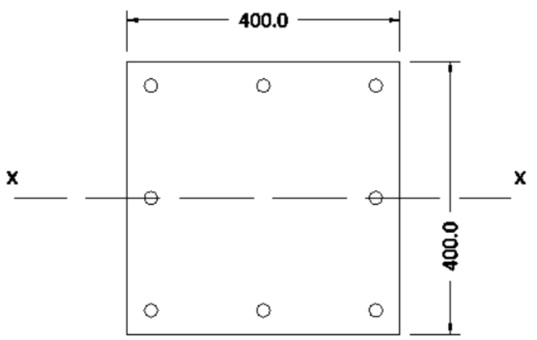
Take the random three design loads from Design Report in Interactive Design and design based on N and M22.

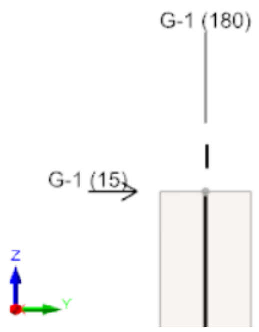
No	N (kN)	M22 (kN.m)
1 -Top	252.0	0.0
-Bottom	268.1	63.0
2 -Top	180.0	0.0
-Bottom	196.1	45.0
6 -Top	216.0	0.0
-Bottom	229.8	62.6

```
Design Loads:

No              N          M22
              (kN)       (kN.m)
------------------------------
  1         268.13        68.00
  2         196.13        48.73
  6         229.82        66.95
------------------------------
```

Case	Axial Load	Without imperfection and second-order effect)	With imperfection and second-order effect
	N (kN)	M22 (kNm)	**M22 (kNm)**
1 (G+Q)	**268.1**	63.0	**68.00**
2 (G+Q)p1	**196.1**	45.0	**48.73**
6 (G+Q+Ny	**229.8**	62.6	**66.95**





Factor:	Symbol:	Value:
Coefficient – EC2, eqn. 3.15	αcc	N/A, therefore 1.0
Effective Strength – BS8110, fig. 2.1	η:	0.67
Reduction Factor – BS8110, fig. 3.3	λ:	0.9
Reduction Factor – EC2, cl.3.1.7(3)	RF:	N/A, therefore 1.0
Max. Strain of Steel	εc:	0.0035
Steel Modulus	Es:	200,000 N/mm^2
Concrete Modification Factor	γm:	1.5
Rebar Modification Factor	γms:	1.15

Plastic Centroid, yp = 200mm

Calculation:

Case G: Pure Tension from Steel

$$Total\ Axial\ Load, N = -Ast * fyd$$
$$N = -361.9kN$$

Case A: Pure Compression:

$$Total\ Axial\ Load, N = Fsc + Fcc$$
$$N = (361.91 + 2{,}131.88)kN$$
$$N = 2{,}493.79\ kN$$

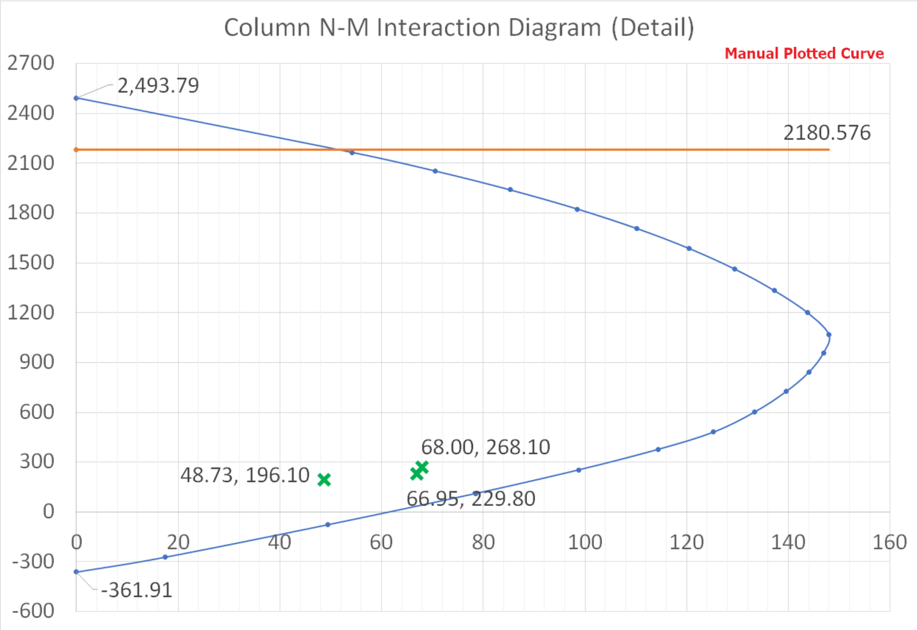
Ultimate Moment Capacity Limit: (BS8110 only)

$$N = 0.4 * fcu * Ac + 0.75 * As * fyd$$

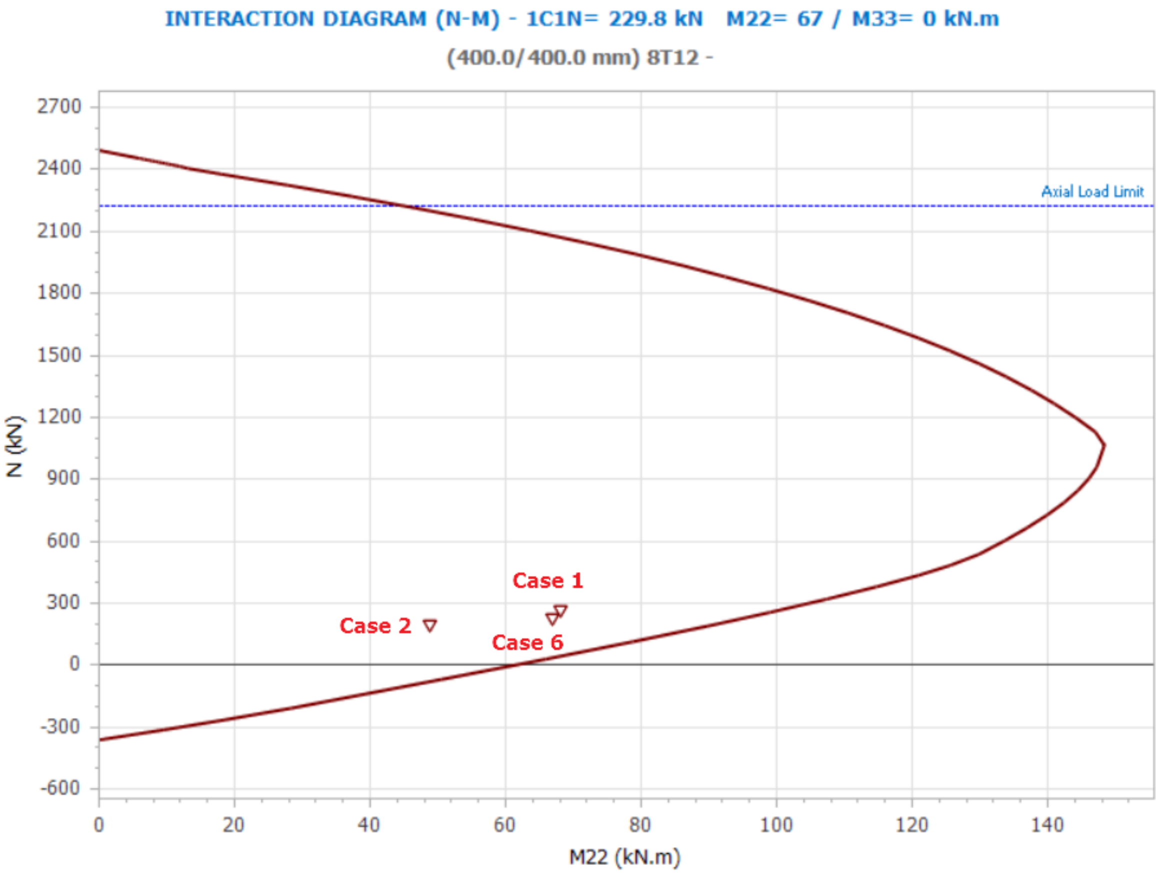
$$N = 0.4 * 30 * 160{,}000 + 0.75 * 904.8 * 400 = 2{,}180.58kN$$

The sample size for x is 20 in this example. Increase the size to have higher precision of the result.

$$x\ (mm) = \begin{pmatrix} 20 \\ 40 \\ 60 \\ 80 \\ 100 \\ 120 \\ 140 \\ 160 \\ 180 \\ 200 \\ 220 \\ 240 \\ 260 \\ 280 \\ 300 \\ 320 \\ 340 \\ 360 \\ 380 \\ 400 \end{pmatrix} N, ult\ (kN) = \begin{pmatrix} -272.217 \\ -75.65 \\ 111.88 \\ 253.88 \\ 377.672 \\ 481.62 \\ 600.72 \\ 725.47 \\ 843.94 \\ 958.01 \\ 1068.89 \\ 1200.27 \\ 1333.84 \\ 1462.12 \\ 1586.16 \\ 1706.75 \\ 1824.51 \\ 1939.9 \\ 2053.3 \\ 2165.01 \end{pmatrix} M, ult\ (kNm) = \begin{pmatrix} 17.38 \\ 49.49 \\ 78.46 \\ 98.69 \\ 114.38 \\ 125.27 \\ 133.28 \\ 139.55 \\ 144.09 \\ 146.88 \\ 147.95 \\ 143.74 \\ 137.18 \\ 129.48 \\ 120.51 \\ 110.19 \\ 98.45 \\ 85.24 \\ 70.51 \\ 54.24 \end{pmatrix}$$



The curve for direction 3 (M22) about "plane X" from ProtaStructure is shown below:

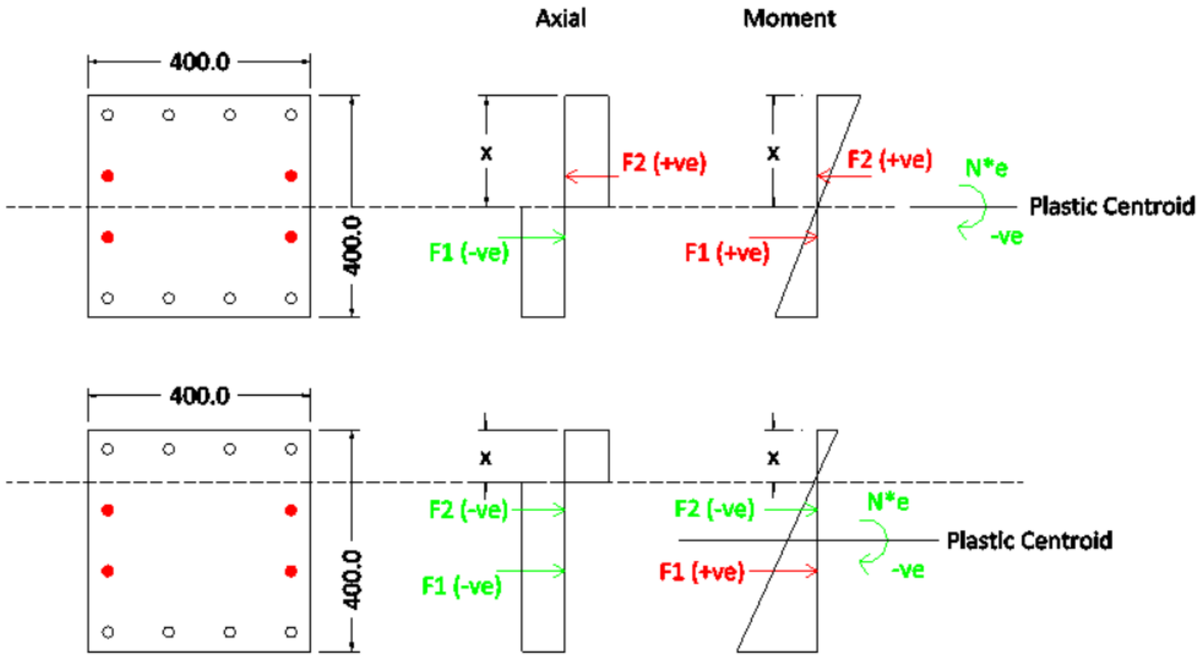


Based on the output from the manually potted curve and the result generated from ProtaStructure, they are proved to be identical. Hence, we can conclude that the N-M Interaction Diagram results in ProtaStructure are correct and adequate.

Intermediate Rebars Capacity

Intermediate rebars provide additional axial load capacity and moment resistance to the column/wall.

The capacity of these bars must be calculated separately and manually added to the total column axial load and Moment capacity. The formula is straightforward.



Formula:

The stress on single intermediate rebar:

$$\sigma s_{,mid} = E_s . \frac{\varepsilon_c . (x - d_{mid})}{x} \qquad but\ -f_{yd} \leq \sigma s_{,mid} \leq f_{yd}$$

(-ve = tension, +ve = compression)

The axial load capacity on single intermediate rebar:

$$Fs_{,mid} = \sigma s_{,mid} . As_{,mid}$$

Moment capacity of single intermediate rebar:

$$M_{,mid} = F_{s,mid} \ (yp - d_{,mid})$$

The calculated axial load capacity and moment capacity from the intermediate rebar is then summed into the column axial load capacity and moment capacity (which has not yet accounted for intermediate rebar capacity) to become the final column capacity.

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